acoustic medium. The dash-dot curves correspond to the asymptotic approximation of the fluid reaction (3.4).

Unlike the acoustic medium, where the reaction on the sphere is produced by the incident wave front for $\tau = \lambda$, the reaction on the sphere in a compressible viscous fluid is non-zero for $\tau > 0$. For $\lambda = 2$, as the viscosity increases (Fig.1), the reaction of the compressible viscous fluid on the sphere progressively deviates from the reaction of the acoustic medium. The amplitude of the reaction of the fluid on the sphere noticeable decreases with distance from the perturbation source (Fig.2) and becomes smoother. The numerical results agree closely with the asymptotic approximation (3.4) for moderate dimensionless times τ.

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ON THE IMPOSSIBILITY OF REGULAR REFLECTION OF A STEADY-STATE SHOCK WAVE FROM THE AXIS OF SYMMETRY*

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Some problems concerning the explanation of the fact that regular reflection of a shock wave from the axis of symmetry is impossible are considered. This fact is well-known and can be demonstrated by linear analysis; it was proved in /1/ by integrating the compatibility condition along the characteristic reaching the point of alleged regular reflection. In this paper, we investigate the flow in the neighbourhood of this point and show that it should be conical. We also prove that the inverse problem of constructing the flow field and the boundary streamline from a given shock wave of arbitrary shape is physically unrealizable in a small neighbourhood of the axis of symmetry.

This topic is also relevant because the literature contains conflicting statements claiming that regular reflection is possible and (much more seldom) impossible, never offering a detailed explanation (see, e.g., /2, 3/). This may explains why this topic has not been treated in detail in authoritative monographs, unlike the similar problem of the collapse of an unsteady-state spherical or cylindrical shock wave.

1. Consider the following proposed picture of the supersonic axisymmetric flow of an

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ideal (non-viscous and non-heat-conducting) gas. In Fig.1, aO and Ob are the incident and the reflected shock waves. A supersonic uniform flow parallel to the x-axis (y is the radial coordinate) impinges on aO from the left. The uniformity assumption does not limit the generality of our treatment, because for a non-uniform incident flow the analysis an be restricted to a sufficiently small neighbourhood of the point $O\ (0, 0)$. On the right side of aO, the slope of the velocity vector is strictly negative (it can be shown by linear analysis that the wave aO does not degenerate into a characteristic as we approach the point O). Continuous supersonic flow is assumed between aO and Ob.

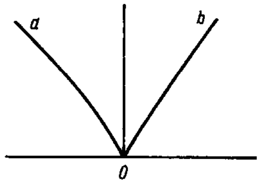

Fig.1

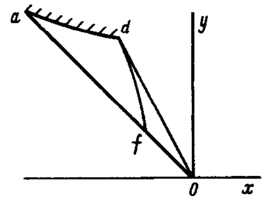

Fig.2

Unlike the plane case, this assumed flow is impossible. In order to elucidate in more detail the reasons for the impossibility of this flow, we will investigate the structure of the assumed flow in a small neighbourhood of the point O. We will write the equations of motion in a polar coordinate system $\lambda, r\ (\lambda = x/y,\ r^2 = x^2 + y^2)$, using as the functions the pressure p and the slope θ of the velocity vector to the x-axis /4/:

$$\begin{gathered}\theta_\lambda (\lambda + k) - p_\lambda (M^2 - 1)(1 - \lambda k) / (\rho q^2) - k = \\ (r\theta_r (1 - \lambda k) + rp_r (M^2 - 1)(\lambda + k) / (\rho q^2))\, y^2 r^{-2} \\ \theta_\lambda (1 - \lambda k) - p_\lambda (\lambda + k) / (\rho q^2) = \\ -(r\theta_r (\lambda + k) + rp_r (1 - \lambda k) / (\rho q^2))\, y^2 r^{-2}; \ k = \operatorname{tg} \theta\end{gathered} \tag{1.1}$$

Here ρ is density, q is the absolute value of the velocity vector, and M is the Mach number. In the region aOb, ρ, q and M depend on p and on the entropy s, which is constant along the streamlines. As we approach the point O, s tends to a finite limit s_0. Therefore, in a small neighbourhood of O, ρ, q and M are known functions of p and s_0 up to $\delta \sim s - s_0$. In Eqs.(1.1), the expressions containing the derivatives with respect to r are collected on the right-hand sides. It is important that θ_r and p_r are always multiplied by r. As a result, the right-hand sides of (1.1) tend to zero as the point O is approached. Indeed, p and θ should have finite values at the point O. Therefore, even if there is a singularity at the point O, the derivatives p_r and θ_r may not increase faster than $r^{-1+\varepsilon}, \varepsilon > 0$, whence it follows that rp_r and $r\theta_r$, and with them the right-hand sides of Eqs.(1.1), tend to zero as we approach the point O.

It follows from the above that, in the assumed flow scheme, Eqs.(1.1) describe conical flow in an infinitely small neighbourhood of the point O. More precisely, p and θ should vary as in a conical flow when we move to the right from aO along the arc of a circle of infinitely small radius centred at the point O.

This conical flow, contiguous along the straight shock wave aO with the uniform flow incident from the left in the direction of the x-axis (Figl.2), was considered in /5/. It was shown that such conical flow may exist in the region afd, where fd is the second-family characteristic tangent to the ray Od at the point d. At the point d we have $\theta_\lambda = -p_\lambda = \infty$. Numerical examples /5/ indicate that rarefaction flow is observed in afd, i.e., $p_\lambda < 0$, and also $\theta_\lambda > 0$.

2. It can be shown that the above discussion is quite general. Indeed, from Eqs.(1.1) with zero right-hand sides we obtain

$$\begin{gathered}\theta_\lambda = -\sin\theta \sin\varphi \sin^2\alpha \cos(\varphi - \theta) / (S^- S^+) \\ p_\lambda / (\rho q^2) = \theta_\lambda \operatorname{tg}(\varphi - \theta) = -\sin\theta \sin\varphi \sin^2\alpha \sin(\varphi - \theta) / (S^- S^+) \\ \varphi = \operatorname{arcctg} \lambda,\ \alpha = \arcsin(M^{-1}),\ S^\pm = \sin(\varphi - \theta \pm \alpha)\end{gathered} \tag{2.1}$$

Behind the shock wave at the point a

$$\theta < 0,\ \cos(\varphi - \theta) < 0,\ S^- > 0;\ \varphi - \theta + \alpha > \pi,\ S^+ < 0$$

The last two inequalities follow from Zemplen's theorem. Therefore, $\theta_\lambda < 0,\ p_\lambda < 0$ for $\lambda = \lambda_a$.

It can be shown that as we move to the right from the point a, i.e., as λ increases, θ, θ_λ, and p_λ do not vanish. Indeed, from $\theta = 0$ for $S^+ < 0$ it follows that the flow should be uniform with $\theta = \theta_\lambda = p_\lambda = 0$ in the entire range of λ, which is impossible. The case when $\varphi + \alpha = \pi$ for $\theta = 0$ was studied in connection with Busemann's conical flows /6/. We merely note that $0/0$ singularities on the right-hand sides of (2.1) may be resolved by l'Hopital's rule, producing the inequalities $\theta_\lambda < 0, p_\lambda > 0$ for this point, which is also impossible for flows originating from the shock waves, behind which we have $\theta < 0$, $\theta_\lambda > 0$, $p_\lambda < 0$.

Thus, for admissible values of λ $(\lambda_a < \lambda < \lambda_d)$, the flow is a rarefaction flow with $p_\lambda < 0$ in which $\theta < 0, \theta_\lambda > 0$. At the point d, we have $\varphi - \theta + \alpha = \pi$, i.e., the second-family characteristic df is tangent to the ray Od. At this point $\theta_\lambda = \infty, p_\lambda = -\infty$. This degeneration is similar to that considered previously /7/ for a different conical rarefaction flow.

Let us estimate the extent of the region of existence of this flow, i.e., the value of $\varphi_a - \varphi_d$. From $p_\lambda < 0$ it follows that $\alpha_\lambda < 0$, and therefore $\theta_\lambda - \alpha_\lambda > 0$. Hence $\varphi_d = \pi + \theta_d - \alpha_d > \pi + \theta_a - \alpha_a$. On the other hand, $\varphi_a < \pi - \alpha_0$ (α_0 is the Mach angle in the incident flow). Thus, $\varphi_a - \varphi_d < \alpha_a - \theta_a - \alpha_0$. Therefore, as the shock wave intensity falls off, we have $\theta_a \to 0$, $\varphi_d \to \varphi_a$.

We should note at this point that, despite the similarity in boundary conditions, this flow is fundamentally different from the Busemann flow originating from a second-family characteristic /6/. Busemann flow, unlike the flow considered above, is a compression flow, it adjoins the symmetry axis and may be transformed by a closing conical shock wave into a flow parallel to the x-axis.

To the right of the characteristic fd, the conical flow does not exist. This can be explained as follows. Take two points f^+ and f^- on the straight shock wave above and below the point f at distances ε^+ and ε^- from it, respectively. The second-family characteristic passing through f^+ intersects the upper wall as the point d^+ with distance δ^+ between d^+ and d. An unlimited increase in the derivatives, θ_λ and p_λ as one approaches the point d implies that $\delta^+ / \varepsilon^+ \to 0$ as $\varepsilon^+ \to 0$. On the other hand, when we formally construct the flow to the right of the characteristic fd and the segment f^-f of the straight shock wave, the second-family characteristic originating from the point f^- will intersect fd below the upper wall for an arbitrarily small ε^-. This flow is therefore physically unrealizable. It also follows that, for any channel profile, the shock wave intensity can only increase to the right of the point d, and the velocity behind the shock wave decreases to subsonic with subsequent flow restructuring and the appearance of irregular reflection.

3. The picture described above is obviously observed also in the more general case of flows with a curvilinear shock wave. This case is conveniently considered in terms of the following inverse problem. Suppose we are given some fairly smooth shock wave aO (Fig.3). On its right side, $\theta < 0, M > 1$. Consider the problem of constructing the flow field and the channel wall to the right of aO.

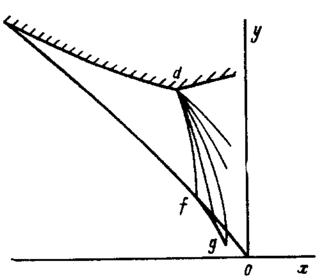

Fig.3

Using the distributions of p and θ along the right side of aO and the equations of gas dynamics or the compatibility conditions, we calculate the derivative $\omega_l = d\omega/dl$, $\omega = \theta - \alpha$, along the second-family characteristic. This derivative in our case is interesting in that it is directly related to the radius of curvature R of the second-family characteristic originating from the given point of the wave aO:

$$\omega_l = \frac{\sin\theta \sin^2\alpha (\cos\beta - \alpha_p \rho q^2 \sin\beta) + yz}{y \sin(\beta - \alpha)} \tag{3.1}$$

$$\alpha_p = \partial\alpha(p, s)/\partial p, \quad R = 1/\omega_l$$

where β $(\beta < 0)$ is the angle between the velocity vector and the shock wave, and z is the combination of the derivatives of p and θ calculated along aO.

As in the analysis of system (1.1), we can show that the product yz from (3.1) tends to zero as $y \to 0$. Seeing that $\sin\theta < 0, \sin\beta < 0$, $\sin(\beta - \alpha) < 0$, $\alpha_p > 0$, we conclude that for $y \to 0$, ω_l increases as $1/y$. This is naturally consistent with the fact that system (1.1) describes conical flow as $y \to 0$.

Thus, as we approach the point O along aO, the slope of the characteristics tends to a finite limit, but the radius of curvature $R = 1/\omega_l$ decreases in proportion to y. The characteristics originating from the points of aO turn counterclockwise.

This leads to the following conclusion. In a sufficiently small neighbourhood of the point O, two near second-family characteristics originating from the points of aO intersect below the channel wall. This implies that the inverse problem of constructing the flow field and the channel wall from a given shock wave is physically unrealizable near the axis of symmetry.

The general flow picture is described as follows. At a sufficient distance from the axis of symmetry, a solution corresponding to the given shock wave aO exists up to the characteristic fd such that, at the point d $\theta_\tau = \infty$ and $p_\tau = -\infty$ (θ_τ and p_τ are derivatives along a streamline). To the right of fd, this problem is unsolvable. For any shape of the wall to the right of the point d, including a bend at the point d (Fig.3), the intensity of the shock wave fg can only increase, with the velocity behind the shock wave decreasing to a subsonic value. This ultimately leads to irregular reflection and the formation of a Mach disk. Numerical results for jets /3/ show that the size of the Mach disk decreases rapidly as the unrated conditions of the jet approach 1. This is consistent with the numeriaal analysis of weakly converging channels with a straight generator /8/. Our argument may provide one of the explanations of the experiments /9/ in which the Mach disk is not detected visually.

In conclusion we recall that in this paper our analysis of the impossibility of regular reflection is based on the boundedness of the gas-dynamic parameters behind the shock wave as it approaches the axis of symmetry. At the same time, no such restrictions are imposed in similar problems with shock wave collapse and reflection in one-dimensional unsteady-state gas dynamics with cylindrical or spherical symmetry. Therefore, in the equations describing wave collapse and reflection in the variables r and $\lambda = t/r$, where r is the radial coordinate and t is the time, we cannot eliminate terms containing derivatives with respect to r. Indeed, in the known solutions of Gooderley, Landau and Stanyukovich /10/, certain parameters, such as pressure and shock wave velocity, increase without limit in the case of collapse. Yet the same problem with additional constraints on the increase in parametrs is unsolvable, becuase as the shock wave approaches the axis of symmetry or point of symmetry, its intensity increases without limit for any motion of the piston. This also highlights a certain analogy with the topics considered in this paper.

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